Design and Analysis of A 6-DOF Haptic Device Using the Singular-Free Parallel Mechanism

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Abstract

This work concerns the design and development of a six degree of freedom Haptic device. The Haptic device is used as the master arm in the teleoperation purposes. For the experimentation, the stewart platform, other parallel mechanism, is used as the slave arm. The force reflex from the slave arm is transmitted through Tendon-Pulley train of the master arm. The virtual guidance can be implemented in the Master arm to improve the maneuverability for the operators as well as the low inertia, friction force, and backlash are minimized in the design. The singularity is also eliminated. The invert, forward kinematics, and Jacobian of the master arm is derived. The closed-form solution of the forward kinematics can be obtained by installing 9 encoders, instead of 6. The end-point of the master arm is sent to the slave arm. The slave arm will transform to the end-point to it joint variables by using the invert kinematics of the slave arm. The experimental results show that the Haptic device is capable of controlling the stewart plate form and can be used for other parallel mechanisms as well.

1) Introduction

Conventional robot arms are typically controlled by preprogramming of the desired paths within the robot workspace. This will limit the capability of the robot arm. There are some applications which we cannot form the desired paths at the beginning. The manual manipulation is needed to help the operator to control the manipulator arm. In this case the master-slave or teleoperation is required to fulfill the task. Besides the path following control of the teleoperation, the information of the force react at the slave should be reflected back to the operator at the master side. This is the Haptic system characteristic. In this work we developed a 6 degree of freedom Master-Slave Human-Assisted manipulator arm with force reflected system or a 6 DOF Haptic system as shown in the figure 1. The mechanism is a singular-free parallel mechanism. The relaxation of kinematics similarity between the master and slave is assumed. We also introduce virtual guidance to improve the maneuverability for the operators. The virtual guidance can be a desired path or a boundary of the desired workspace. And any limited control volume can also be controlled or specified by virtual guidance technique.

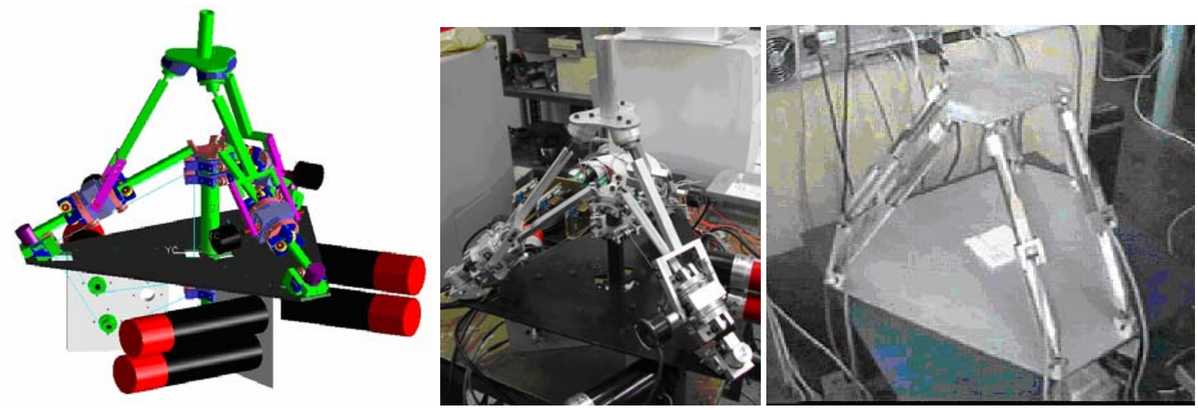


Figure 1. The 6 DOF Haptic System as the Master and the Stewart as the Slave

2) The Master-Slave System

Figure 1 shows the Master-Slave arm developed for this project. The master arm is a 6 DOF with tendon-pulley train driving mechanism. The mechanism is back drivable. The tendon-pulley system used in this mechanism has the same functions as the pulley-belt with fixed or variable distance between the two unparallel rotating axes. As shown in figure 2, for the sliding-rotating link, we introduce a unique design of the tendon-pulley system which supports decouple motion of translation and rotation within the same driving mechanism.

Forward Kinematics

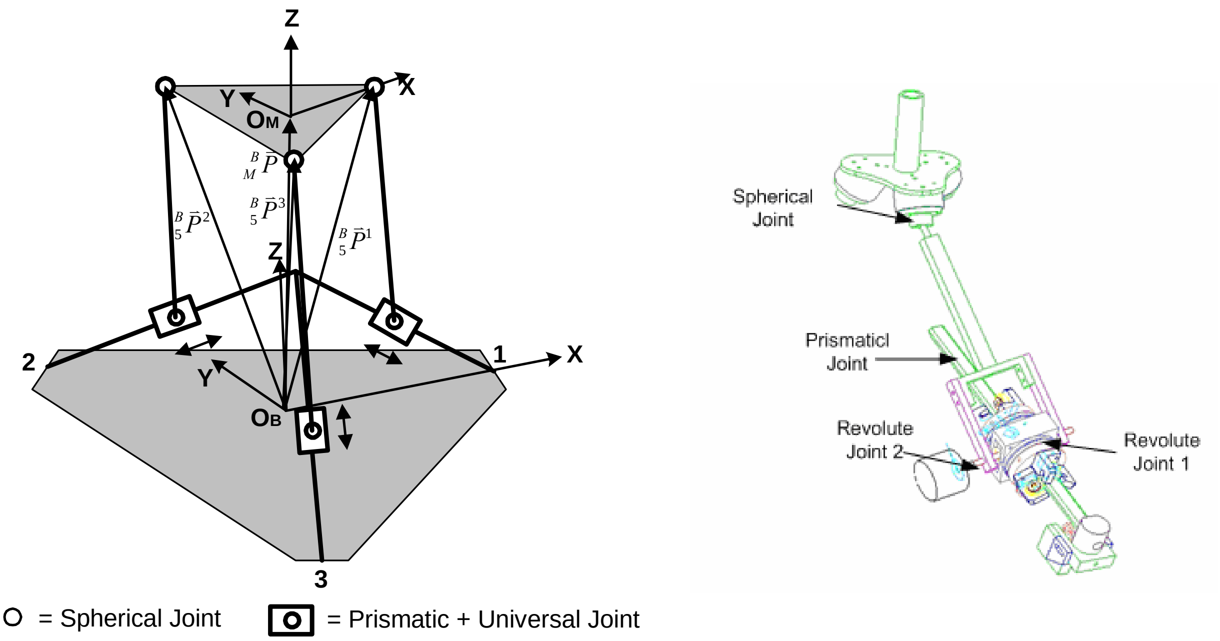


Figure 2 Define the position vector of each joint 5 with respect to the base frame and illustrated the Tendon-Pulley system of the sliding-rotating link of each link

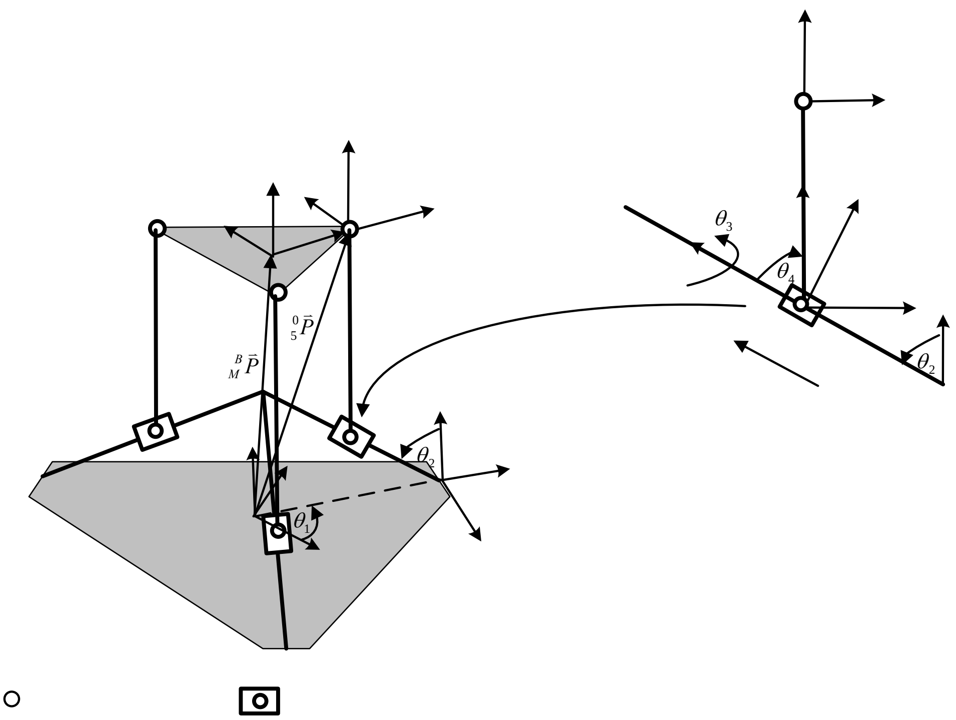


Figure 3. The Configuration of the Coordinate Frame of the Haptic Arm

From figure 3, for each link, i = 1, 2, 3. the r_b represents the distance between the center of the base (frame 0 or frame B) and center of the link or the origin of the link coordinate frame (frame i),

d^i represents the distance of the sliding joint of link i, and L^i is the length of link i as shown in the figure. ($L^1 = L^2 = L^3 = L$). From the figure, it can be shown that $\theta_1^1 = 0°$, $\theta_1^2 = 120°$, $\theta_1^3 = 240°$ $\theta_2^1 = \theta_2^2 = \theta_2^3 = 60°$. And we will define that $c = \cos$ and $s = \sin$. From figure 3, the forward kinematics can be derived starting for base frame (frame 0 or frame B). Frame 1 is the origin of the prismatic joint and frame 2 is attached to the origin of the sliding joint which moved in the direction of the prismatic joint. The origin of frame 3 and frame 4 are located at the same position of the origin of the frame 2 and attached to the rotating joints 1 and 2 respectively as illustrated in the figure 2. And frame 5 is attached to the spherical joint. Figure 3 shows the configuration of all the coordinate system. The homogeneous transformation of the coordinate frame j to the coordinate frame j+1 of link i can be written as following ($j = 0,1,2,3,4$ and frame 0 is frame B)

$$ {}_1^BT^i = \begin{bmatrix} c\theta_1^i & 0 & s\theta_1^i & r_b c\theta_1^i \\ s\theta_1^i & 0 & -c\theta_1^i & r_b s\theta_1^i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_2^1T^i = \begin{bmatrix} c\theta_2^i & 0 & -s\theta_2^i & -d^i s\theta_2^i \\ s\theta_2^i & 0 & c\theta_2^i & d^i c\theta_2^i \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_3^2T^i = \begin{bmatrix} c\theta_3^i & -s\theta_3^i & 0 & 0 \\ s\theta_3^i & c\theta_3^i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_4^3T^i = \begin{bmatrix} s\theta_4^i & c\theta_4^i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ c\theta_4^i & -s\theta_4^i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}_5^4T^i = \begin{bmatrix} 1 & 0 & 0 & L \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

So, the total transformation matrix of frame B to frame 5 of link i can be written as ${}_5^BT^i = {}_1^BT^i \times {}_2^1T^i \times {}_3^2T^i \times {}_4^3T^i \times {}_5^4T^i$, or

$$ {}_5^BT^i = \begin{bmatrix} {}_5^BT_{1,1}^i & {}_5^BT_{1,2}^i & {}_5^BT_{1,3}^i & {}_5^BT_{1,4}^i \\ {}_5^BT_{2,1}^i & {}_5^BT_{2,2}^i & {}_5^BT_{2,3}^i & {}_5^BT_{2,4}^i \\ {}_5^BT_{3,1}^i & {}_5^BT_{3,2}^i & {}_5^BT_{3,3}^i & {}_5^BT_{3,4}^i \\ {}_5^BT_{4,1}^i & {}_5^BT_{4,2}^i & {}_5^BT_{4,3}^i & {}_5^BT_{4,4}^i \end{bmatrix} \qquad (1) $$

$$ \begin{aligned}
{}_5^BT_{1,1}^i &= \left(c\theta_1^i c\theta_2^i c\theta_3^i - s\theta_1^i s\theta_3^i\right)s\theta_4^i - c\theta_1^i s\theta_2^i c\theta_4^i \\
{}_5^BT_{2,1}^i &= \left(s\theta_1^i c\theta_2^i c\theta_3^i + c\theta_1^i s\theta_3^i\right)s\theta_4^i - s\theta_1^i s\theta_2^i c\theta_4^i \\
{}_5^BT_{3,1}^i &= s\theta_2^i c\theta_3^i s\theta_4^i + c\theta_2^i c\theta_4^i \\
{}_5^BT_{4,1}^i &= 0 \\
{}_5^BT_{1,2}^i &= \left(c\theta_1^i c\theta_2^i c\theta_3^i - s\theta_1^i s\theta_3^i\right)c\theta_4^i + c\theta_1^i s\theta_2^i s\theta_4^i \\
{}_5^BT_{2,2}^i &= \left(s\theta_1^i c\theta_2^i c\theta_3^i + c\theta_1^i s\theta_3^i\right)c\theta_4^i + s\theta_1^i s\theta_2^i s\theta_4^i \\
{}_5^BT_{3,2}^i &= s\theta_2^i c\theta_3^i c\theta_4^i - c\theta_2^i s\theta_4^i \\
{}_5^BT_{4,2}^i &= 0 \\
{}_5^BT_{1,3}^i &= -c\theta_1^i c\theta_2^i s\theta_3^i - s\theta_1^i c\theta_3^i \\
{}_5^BT_{2,3}^i &= -s\theta_1^i c\theta_2^i s\theta_3^i + c\theta_1^i c\theta_3^i \\
{}_5^BT_{3,3}^i &= -s\theta_2^i s\theta_3^i \\
{}_5^BT_{4,3}^i &= 0 \\
{}_5^BT_{1,4}^i &= \left(\left(c\theta_1^i c\theta_2^i c\theta_3^i - s\theta_1^i s\theta_3^i\right)s\theta_4^i - c\theta_1^i s\theta_2^i c\theta_4^i\right)L - d^i\left(c\theta_1^i s\theta_2^i\right) + r_b c\theta_1^i \\
{}_5^BT_{2,4}^i &= \left(\left(s\theta_1^i c\theta_2^i c\theta_3^i + c\theta_1^i s\theta_3^i\right)s\theta_4^i - s\theta_1^i s\theta_2^i c\theta_4^i\right)L - d^i\left(s\theta_1^i s\theta_2^i\right) + r_b s\theta_1^i \\
{}_5^BT_{3,4}^i &= \left(s\theta_2^i c\theta_3^i s\theta_4^i + c\theta_2^i c\theta_4^i\right)L + d^i\left(c\theta_2^i\right) \\
{}_5^BT_{4,4}^i &= 1
\end{aligned} $$

So, the position vector of the coordinate frame 5 with respect to coordinate frame B of link i can be written as

$$\begin{bmatrix} {}^{B}_{5}\vec{P}^{i} \\ 1 \end{bmatrix} = {}^{B}_{5}T^{i} \times \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \left(\left(c\theta_1^i c\theta_2^i c\theta_3^i - s\theta_1^i s\theta_3^i\right)s\theta_4^i - c\theta_1^i s\theta_2^i c\theta_4^i\right)L - d^i\left(c\theta_1^i s\theta_2^i\right) + r_b c\theta_1^i \\ \left(\left(s\theta_1^i c\theta_2^i c\theta_3^i + c\theta_1^i s\theta_3^i\right)s\theta_4^i - s\theta_1^i s\theta_2^i c\theta_4^i\right)L - d^i\left(s\theta_1^i s\theta_2^i\right) + r_b s\theta_1^i \\ \left(s\theta_2^i c\theta_3^i s\theta_4^i + c\theta_2^i c\theta_4^i\right)L + d^i\left(c\theta_2^i\right) \\ 1 \end{bmatrix}$$

Figure 2 shown the vector position with respect to base frame and the corresponding homogeneous transformation can be written as

$${}^{B}_{M}T = \begin{bmatrix} \vec{n} & \vec{o} & \vec{a} & {}^{B}_{M}\vec{P} \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{where the position vector is} \quad {}^{B}_{M}\vec{P} = \frac{1}{3}\left({}^{B}_{5}\vec{P}^{1} + {}^{B}_{5}\vec{P}^{2} + {}^{B}_{5}\vec{P}^{3}\right)$$

And the Orientation of frame M with respect to frame B is

$$\vec{n} = \frac{{}^{B}_{5}\vec{P}^{1} - {}^{B}_{M}\vec{P}}{\left|{}^{B}_{5}\vec{P}^{1} - {}^{B}_{M}\vec{P}\right|}, \qquad \vec{o} = \frac{{}^{B}_{5}\vec{P}^{2} - {}^{B}_{5}\vec{P}^{3}}{\left|{}^{B}_{5}\vec{P}^{2} - {}^{B}_{5}\vec{P}^{3}\right|}, \qquad \vec{a} = \vec{n} \times \vec{o}$$

Invert Kinematics

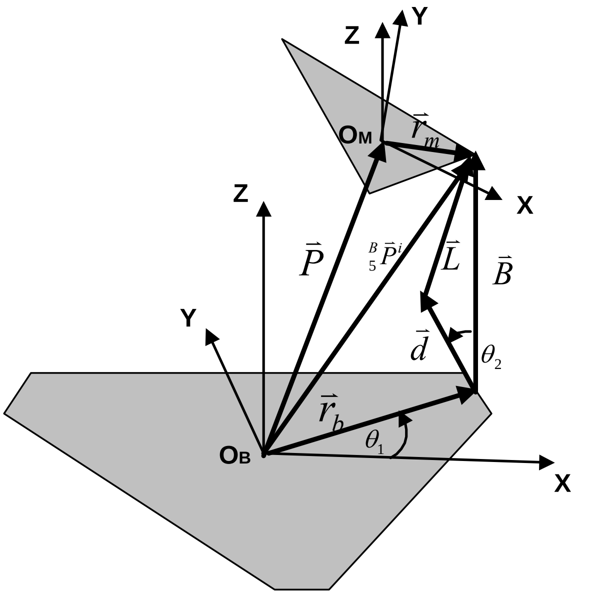


Figure 4. The Necessary Vectors Needed for Deriving Invert Kinematics.

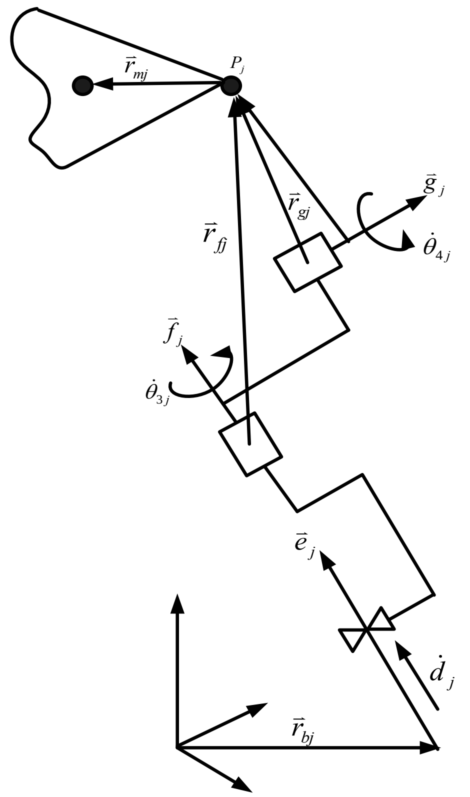


Figure 5. The Position Vector of Link j

Figure 4 defined the position vectors needed for the invert kinematics as following:

Position vector $\vec{P} = \begin{bmatrix} X & Y & Z \end{bmatrix}^T$ represents the vector form the origin O_B to the endpoint origin O_M

For link i, $\vec{r}_m^{\,i} = \begin{bmatrix} r_m c\theta_1^i & r_m s\theta_1^i & 0 \end{bmatrix}^T$, $\vec{r}_b^{\,i} = \begin{bmatrix} r_b c\theta_1^i & r_b s\theta_1^i & 0 \end{bmatrix}^T$, the sliding position

$\vec{d}^i = \begin{bmatrix} d_x^i & d_y^i & d_z^i \end{bmatrix}^T = d^i \vec{d}_u^i$ where $d^i = \begin{bmatrix} d^i & 0 & 0 \\ 0 & d^i & 0 \\ 0 & 0 & d^i \end{bmatrix}$ and $\vec{d}_u^i = \begin{bmatrix} -c\theta_1^i s\theta_2^i & -s\theta_1^i s\theta_2^i & c\theta_2^i \end{bmatrix}^T$ (represent the unit vector of $\vec{d}^i$). $\vec{L}^i = \begin{bmatrix} L_x^i & L_y^i & L_z^i \end{bmatrix}^T$ represents the vector of link i.

The desired joint variables, $\theta_3^i, \theta_4^i, d^i$, can be derived from the equation below
$\vec{B}^i = \vec{P} + {}_M^B R \vec{r}_m^i - \vec{r}_b^i$ and from ${}_5^B\vec{P}^i = \vec{P} + {}_M^B R \vec{r}_m^i = {}_M^B T \vec{r}_m^i$, so, $\vec{B}^i = {}_M^B T \vec{r}_m^i - \vec{r}_b^i$
Where ${}_M^B R = \begin{bmatrix} \vec{n} & \vec{o} & \vec{a} \end{bmatrix} = R_{RPY}$ represent the Roll-Pitch-Yaw orientation, R_{RPY}, which is equal to

$$R_{RPY} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$ where α = Roll , β = Pitch , γ = Yaw

So, $\vec{B}^i$ is equal to

$$\begin{bmatrix} \vec{B}^i \\ 1 \end{bmatrix} = \begin{bmatrix} B_x^i \\ B_y^i \\ B_z^i \\ 1 \end{bmatrix} = {}_m^B T \begin{bmatrix} r_m c\theta_1^i \\ r_m s\theta_1^i \\ 0 \\ 1 \end{bmatrix} - \begin{bmatrix} r_b c\theta_1^i \\ r_b s\theta_1^i \\ 0 \\ 1 \end{bmatrix}$$

where ${}_M^B T$ is derived previously, so B_x^i, B_y^i, B_z^i can be written as

$$B_x^i = \left(\left(c\theta_1^i c\theta_2^i c\theta_3^i - s\theta_1^i s\theta_3^i \right) s\theta_4^i - c\theta_1^i s\theta_2^i c\theta_4^i \right) L - d^i \left(c\theta_1^i s\theta_2^i \right) \quad (2)$$

$$B_y^i = \left(\left(s\theta_1^i c\theta_2^i c\theta_3^i + c\theta_1^i s\theta_3^i \right) s\theta_4^i - s\theta_1^i s\theta_2^i c\theta_4^i \right) L - d^i \left(s\theta_1^i s\theta_2^i \right) \quad (3)$$

$$B_z^i = \left(s\theta_2^i c\theta_3^i s\theta_4^i + c\theta_2^i c\theta_4^i \right) L + d^i \left(c\theta_2^i \right) \quad (4)$$

Equation (2) – (4) is used to solve θ_3^i as following: multiply both sides of equation (3) and (4) with $c\theta_2^i$ and $s\theta_1^i s\theta_2^i$ respectively, and then sum them together, we will obtain
$s\theta_1^i s\theta_2^i B_z^i + c\theta_2^i B_y^i = \left(s\theta_1^i c\theta_3^i + c\theta_1^i c\theta_2^i s\theta_3^i \right) s\theta_4^i L$

So, $$s\theta_4^i L = \frac{s\theta_1^i s\theta_2^i B_z^i + c\theta_2^i B_y^i}{s\theta_1^i c\theta_3^i + c\theta_1^i c\theta_2^i s\theta_3^i} \quad (5)$$

Similarly, multiply both side of equation (2) and (4) with $c\theta_2^i$ and $c\theta_1^i s\theta_2^i$ respectively, then sum them together, we will obtain $c\theta_1^i s\theta_2^i B_z^i + c\theta_2^i B_x^i = \left(c\theta_1^i c\theta_3^i - s\theta_1^i c\theta_2^i s\theta_3^i \right) s\theta_4^i L$

So, $$s\theta_4^i L = \frac{c\theta_1^i s\theta_2^i B_z^i + c\theta_2^i B_x^i}{c\theta_1^i c\theta_3^i - s\theta_1^i c\theta_2^i s\theta_3^i} \quad (6)$$

From equation (5) and (6), we will obtain θ_3^i as following

$$\theta_3^i = \arctan\left(\frac{c\theta_1^i c\theta_2^i B_y^i - s\theta_1^i c\theta_2^i B_x^i}{c\theta_2^i s\theta_2^i B_z^i + \left(c\theta_2^i \right)^2 \left(c\theta_1^i B_x^i + s\theta_1^i B_y^i \right)} \right). \quad (7)$$

Similarly, we can obtain

$$\theta_4^i = \arcsin\left(\frac{s\theta_2^i B_z^i + c\theta_2^i\left(c\theta_1^i B_x^i + s\theta_1^i B_y^i\right)}{c\theta_3^i L}\right) \tag{8}$$

And d^i can be derived from equation (4) as $B_z^i = \left(s\theta_2^i c\theta_3^i s\theta_4^i + c\theta_2^i c\theta_4^i\right)L + d^i\left(c\theta_2^i\right)$

So,

$$d^i = \frac{B_z^i - \left(s\theta_2^i c\theta_3^i s\theta_4^i + c\theta_2^i c\theta_4^i\right)L}{c\theta_2^i} \tag{9}$$

Jacobian

The Jacobian is the relationship between the twist velocity of the moving platform and the velocity of the joint variables where the actuators are attached. From the relationship $Bt = A\dot{q}$, the matrix A, B are the Jacobian matrix of the closed-loop chain (parallel mechanism) and $t = \begin{bmatrix}\omega \\ v\end{bmatrix} = \begin{bmatrix}\omega_x & \omega_y & \omega_z & v_x & v_y & v_z\end{bmatrix}^T$ represent the velocity of the top plate which consist of the angular velocity and linear velocity. And $\dot{q} = \begin{bmatrix}\dot{d}_1 & \dot{\theta}_{31} & \dot{d}_2 & \dot{\theta}_{32} & \dot{d}_3 & \dot{\theta}_{33}\end{bmatrix}^T$ represent the velocity of the joint variables where the actuators attached to.

From figure 5, the velocity of point $\dot{\vec{P}}_j$ is $\dot{\vec{P}}_j = \dot{d}\vec{S}_{j1} + \dot{\theta}_3\vec{S}_{j2} + \dot{\theta}_4\vec{S}_{j3}$ (10)

Or $\dot{\vec{P}}_j = \vec{v} - \vec{\omega} \times \vec{r}_{mj}$ (11)

where $\vec{S}_{j1} = \vec{e}_j$, $\vec{S}_{j2} = \vec{f}_j \times \vec{r}_{fj}$, $\vec{S}_{j2} = \vec{g}_j \times \vec{r}_{gj}$, $\vec{r}_{mj} = \vec{P} - \vec{P}_j$

$$\vec{r}_{fj} = \vec{r}_{gj} = \vec{P}_j - {}_4^B\vec{P}_j = \vec{P}_j - {}_3^B\vec{P}_j$$

$$\vec{r}_{fj} = L\begin{bmatrix}\left(c_1c_2c_3 - s_1s_3\right)s_4 - c_1s_2c_4 \\ \left(s_1c_2c_3 + c_1s_3\right)s_4 - s_1s_2c_4 \\ s_2c_3s_4 + c_2c_4\end{bmatrix}; \quad j = 1,2,3$$

And because $\left(\vec{S}_{j2} \times \vec{S}_{j3}\right)^T \cdot \dot{\vec{P}}_j = \dot{d}\left(\vec{S}_{j2} \times \vec{S}_{j3}\right)^T \cdot \vec{S}_{j1}$ (12)

So,

$$\frac{\left(\vec{S}_{j2} \times \vec{S}_{j3}\right)^T}{\left|\vec{S}_{j2} \times \vec{S}_{j3}\right|} \cdot \dot{\vec{P}}_j = \dot{d}\,\frac{\left(\vec{S}_{j2} \times \vec{S}_{j3}\right)^T \cdot \vec{S}_{j1}}{\left|\vec{S}_{j2} \times \vec{S}_{j3}\right|} \tag{13}$$

Given

$J_j = \begin{bmatrix}\vec{S}_{j1} & \vec{S}_{j2} & \vec{S}_{j3}\end{bmatrix}$ has dimension 3x3

$\Delta_j = \det(J_j) = \left(\vec{S}_{j1} \times \vec{S}_{j2}\right)^T \cdot \vec{S}_{j3}$. So,

$\Delta_j = -s_{j4}c_{j4}L^2$ and

$a_{j1} = \dfrac{\Delta_j}{\left|\vec{S}_{j2} \times \vec{S}_{j3}\right|}$ (dimension 1x1). So,

$a_{j1} = -c_{j4}$

$\vec{I}_j = \dfrac{\vec{S}_{j2} \times \vec{S}_{j3}}{\left|\vec{S}_{j2} \times \vec{S}_{j3}\right|}$ is the unit vector parallel to the vector of $\left(\vec{S}_{j2} \times \vec{S}_{j3}\right)$. So,

$$\vec{I}_j = \begin{bmatrix} (s_1 s_3 - c_1 c_2 c_3) s_4 + c_1 s_2 c_4 \\ -(c_1 s_3 + s_1 c_2 c_3) s_4 + s_1 s_2 c_4 \\ -(s_2 c_3 s_4 + c_2 c_4) \end{bmatrix}_j$$

From the above equation with equation (11) and the equation (13), we will obtain

$$a_{j1} \cdot \dot{d} = \vec{I}_j \cdot \left(\vec{v} - \vec{\omega} \times \vec{r}_{mj}\right) \tag{14}$$

Given $b_{j1} = \left[\left(\vec{I}_j \times \vec{r}_{mj}\right)^T \quad \vec{I}_j^{\,T}\right]$ (dimension 1x6). So, from equation (14), we will obtain

$$a_{j1} \cdot \dot{d} = b_{j1} t \tag{15}$$

From equation (10), we will obtain

$$\left(\vec{S}_{j3} \times \vec{S}_{j1}\right)^T \cdot \dot{\vec{P}}_j = \dot{\theta}_{3j} \left(\vec{S}_{j3} \times \vec{S}_{j1}\right)^T \cdot \vec{S}_{j2} \tag{16}$$

So,

$$\frac{\left(\vec{S}_{j3} \times \vec{S}_{j1}\right)^T}{\left|\vec{S}_{j3} \times \vec{S}_{j1}\right|} \cdot \dot{\vec{P}}_j = \dot{\theta}_{3j} \frac{\left(\vec{S}_{j3} \times \vec{S}_{j1}\right)^T \cdot \vec{S}_{j2}}{\left|\vec{S}_{j3} \times \vec{S}_{j1}\right|} \tag{17}$$

Given

$a_{j2} = \dfrac{\Delta_j}{\left|\vec{S}_{j3} \times \vec{S}_{j1}\right|}$ (dimension 1x1). So,

$$a_{j2} = -s_{j4} L$$

$\vec{M}_j = \dfrac{\vec{S}_{j3} \times \vec{S}_{j1}}{\left|\vec{S}_{j3} \times \vec{S}_{j1}\right|}$ is the unit vector parallel to the vector of $\left(\vec{S}_{j3} \times \vec{S}_{j1}\right)$. So,

$$\vec{M}_j = \begin{bmatrix} c_1 c_2 s_3 + s_1 c_3 \\ s_1 c_2 s_3 - c_1 c_3 \\ s_2 s_3 \end{bmatrix}$$

From the above equation with equation (11) and the equation (17), we will obtain

$$a_{j2} \cdot \dot{\theta}_{3j} = \vec{M}_j \cdot \left(\vec{v} - \vec{\omega} \times \vec{r}_{mj}\right) \tag{18}$$

Given $b_{j2} = \left[\left(\vec{M}_j \times \vec{r}_{mj}\right)^T \quad \vec{M}_j^{\,T}\right]$ (dimension 1x6). So, from equation (18), we obtain

$$a_{j2} \cdot \dot{\theta}_{3j} = b_{j2} t$$

So, the matrix A can be written as

$$A=\begin{bmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{32} \end{bmatrix}$$ which is a diagonal matrix. So, A^{-1} can be obtained from

$$A^{-1}=\begin{bmatrix} 1/a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/a_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/a_{21} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/a_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/a_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/a_{32} \end{bmatrix}$$

And the matrix B is equal to

$$B=\begin{bmatrix} b_{11} \\ b_{12} \\ b_{21} \\ b_{22} \\ b_{31} \\ b_{32} \end{bmatrix}=\begin{bmatrix} \left(\vec{I}_1\times\vec{r}_{m1}\right)^T & \vec{I}_1^T \\ \left(\vec{M}_1\times\vec{r}_{m1}\right)^T & \vec{M}_1^T \\ \left(\vec{I}_2\times\vec{r}_{m2}\right)^T & \vec{I}_2^T \\ \left(\vec{M}_2\times\vec{r}_{m2}\right)^T & \vec{M}_2^T \\ \left(\vec{I}_3\times\vec{r}_{m3}\right)^T & \vec{I}_3^T \\ \left(\vec{M}_3\times\vec{r}_{m3}\right)^T & \vec{M}_3^T \end{bmatrix}$$

The relationship of the force at the endpoint of the Haptic system and the joint torque for the parallel mechanism can be written as

$$F=J^T\tau$$

where $F=\begin{bmatrix}M_x & M_y & M_z & F_x & F_y & F_z\end{bmatrix}^T$ is the force apply at the endpoint (top plate) of the Haptic device. And $\tau=\begin{bmatrix}f_1 & \tau_1 & f_2 & \tau_2 & f_3 & \tau_3\end{bmatrix}^T$ is the joint torque of the actuators

So, the Jacobian matrix is

$$J=A^{-1}B$$

So, the singularity of the system will occur at $\det(A)=0$ or $\det(B)=0$ or $\det(A^{-1}B)=0$. It can be shown that, for the purposed mechanism, there is no singularity.

3) The Controller

Figure 6 shows the connection between the master-arm or the Haptic system and the slave-arm (the Stewart platform). The endpoint position and orientation of the Haptic system will be transformed to the joint command of the Stewart platform. The controller handles both position control and force control.

Figure 7 is the diagram to show the master-slave control system. The operator will control or maneuver the master arm though the control stick. The virtual wall (will be explained in next section) will be specified in advance if necessary. When the control stick hits the virtual wall, the reaction force F, which reacts to the operator, will be generated by the controller in the Cartesian space of the Haptic system. The reaction force consists of 2 components, the force normal to the virtual wall and the viscous force generated by the controller to prevent the control stick move to fast. The reaction force F, in Cartesian space, can be transformed into the joint space by using the Jacobian matrix. The friction force compensation is also included in the control system as shown in the figure 7. When the control stick is inside the working area or in the free area, the reaction force applied at the control stick will only consist of the viscous force and friction compensation force. The information from the encoders at each joint of the master arm will be used to calculate the desired path motion, translation and orientation, of the slave. The detail of the kinematics of the slave arm can be consulted from ref [1]. Then the joint variables of each links of the slave arm can be calculated by using the inverse kinematics. The PID control is used in the control loop of the slave arm as shown in figure 7. Then, the position and orientation error of the slave arm are obtained from the comparison of the measurement values with the input or the reference values.

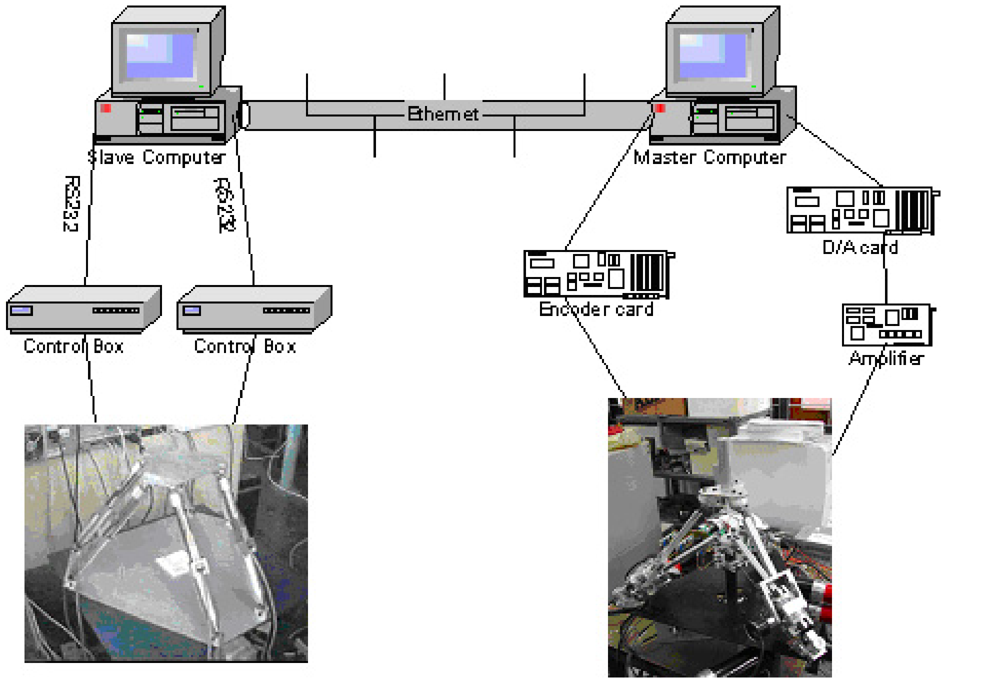


Figure 6. The Connection between the Master-Arm (Haptic) and the Slave-Arm (Stewart)

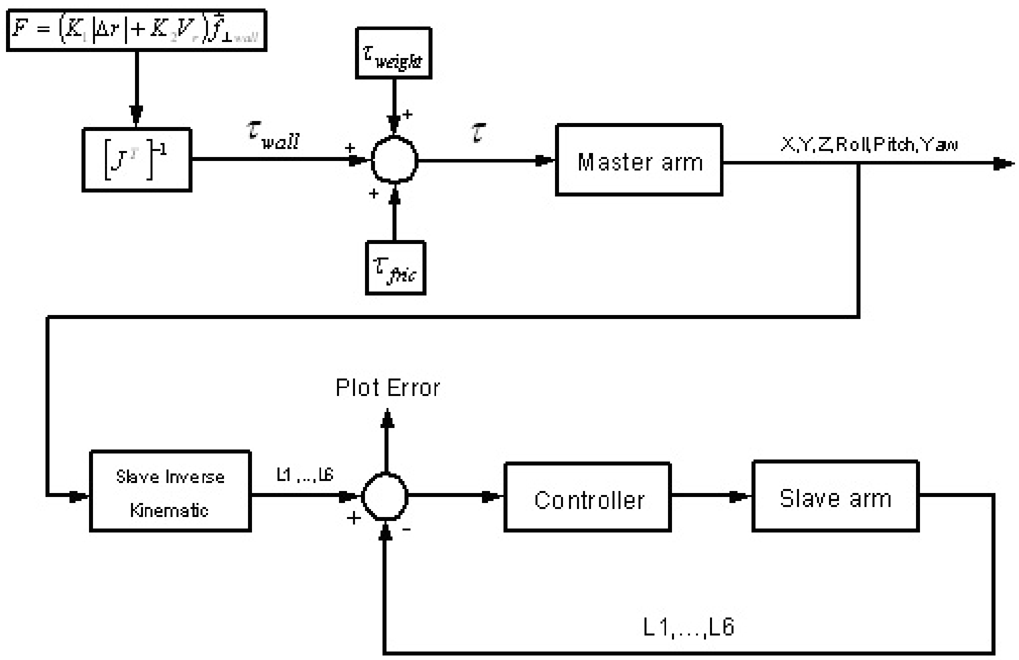


Figure 7 The Structure of the Control System.

4) VIRTUAL WALL

The concept of virtual wall is to specify the working area of the master arm virtually. This will help the operator to work in the specific area more convenience. Figure 8 shows the circular working area. The virtual wall is defined by a function, f(x,y,z) = constant. When the operator move the master arm contact to the virtual wall, the reaction force, F, will be generated to against the operation motion.

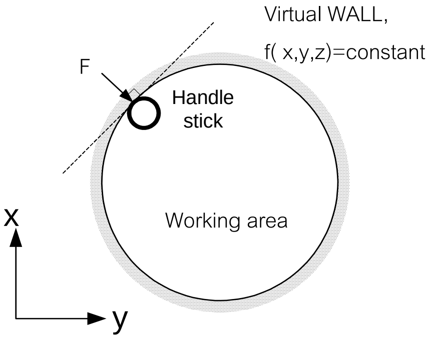


Figure 8. The circular working area

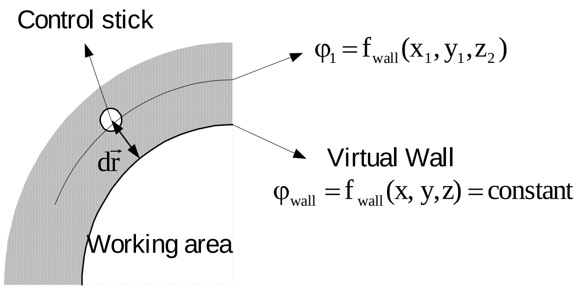


Figure 9.The Control Stick Are Beyond The Virtual Wall

From figure 9, the force F can be evaluated from

$$F = (K_1 dr)\vec{n} + (K_2 V)\vec{n}$$

where

$dr = \dfrac{d\varphi|\nabla\varphi|}{\nabla\varphi\cdot\nabla\varphi}$ = the distance between the control stick and the virtual wall

$\vec{n} = \pm\dfrac{\nabla\varphi}{|\nabla\varphi|}$ = the unit normal vector to the virtual wall

$$\nabla\varphi = i\frac{\partial\varphi}{\partial x} + j\frac{\partial\varphi}{\partial y} + k\frac{\partial\varphi}{\partial z}$$

V = velocity of the control stick

$(K_1 dr)\vec{n}$ = reaction force normal to the wall

$(K_2 V)\vec{n}$ = viscous force generated by the controller

K_1, K_2 = amplifier gains

So, the force F can be written as

$$F = \frac{K_1 d\varphi}{|\nabla\varphi|^2}\nabla\varphi + K_2 V\frac{\nabla\varphi}{|\nabla\varphi|}$$

where

$$\begin{aligned} d\varphi &= f_{wall}(x_1, y_1, z_1) - f_{wall}(x, y, z) \\ &= f_{wall}(x_1, y_1, z_1) - \varphi_{wall} \\ &= f_{wall}(x_1, y_1, z_1) - cons\tan t \end{aligned}$$

5) Experimental Results

Table 1 shows the maximum error measure at the endpoint of the Stewart platform. The experiment is done with 300 Hz., sampling frequency, for the Haptic system and 20 Hz for the stewart platform. The sampling frequencies are limited due to the hardware capability. The error will be much improved if we better controller with higher sampling frequencies.

Figure 10 and 11 are the 3D and 2D diagram, respectively, of the endpoint position of the Haptic arm with circular virtual wall. The center point of the circular virtual wall is at x,y,z = 0. From figure 11, we can estimate the error from the virtual wall is approximately 4 mm. This is due to the limitation of the controller hardware of the Haptic system and the stewart platform. This error can be improved if we can increase the sampling frequencies of both the Haptic system and the stewart platform. Otherwise the digital controller design technique is needed for studying the sampling time effect to the controlled system.

Link i	Error (mm)
1	2.07
2	3.85
3	1.34
4	1.08
5	1.51
6	2.52

Table 1. The Maximum Error of the Endpoint (Top Plate) of the Stewart Platform

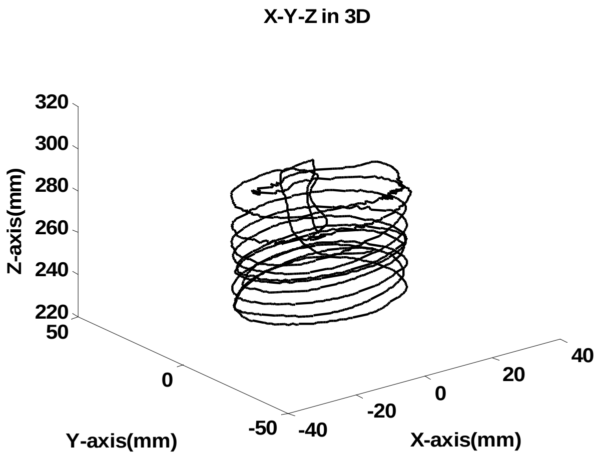
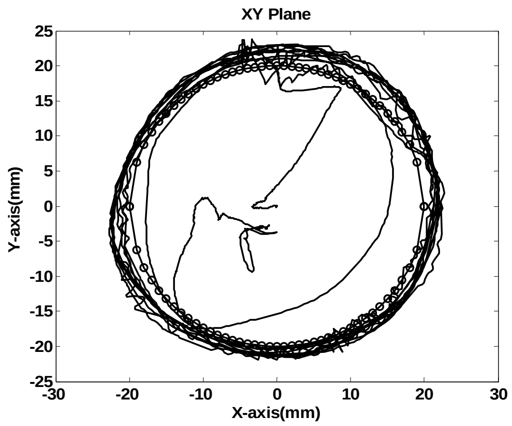


Figure 10. The 3D Diagram of the endpoint of the Haptic Arm with Circular Virtual Wall (Cylindrical Volume)



______ Endpoint position of the Haptic Arm
-o-o-o- Circular Virtual Wall

Figure 11. The 2D Diagram of the endpoint of the Haptic Arm with Circular Virtual Wall

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